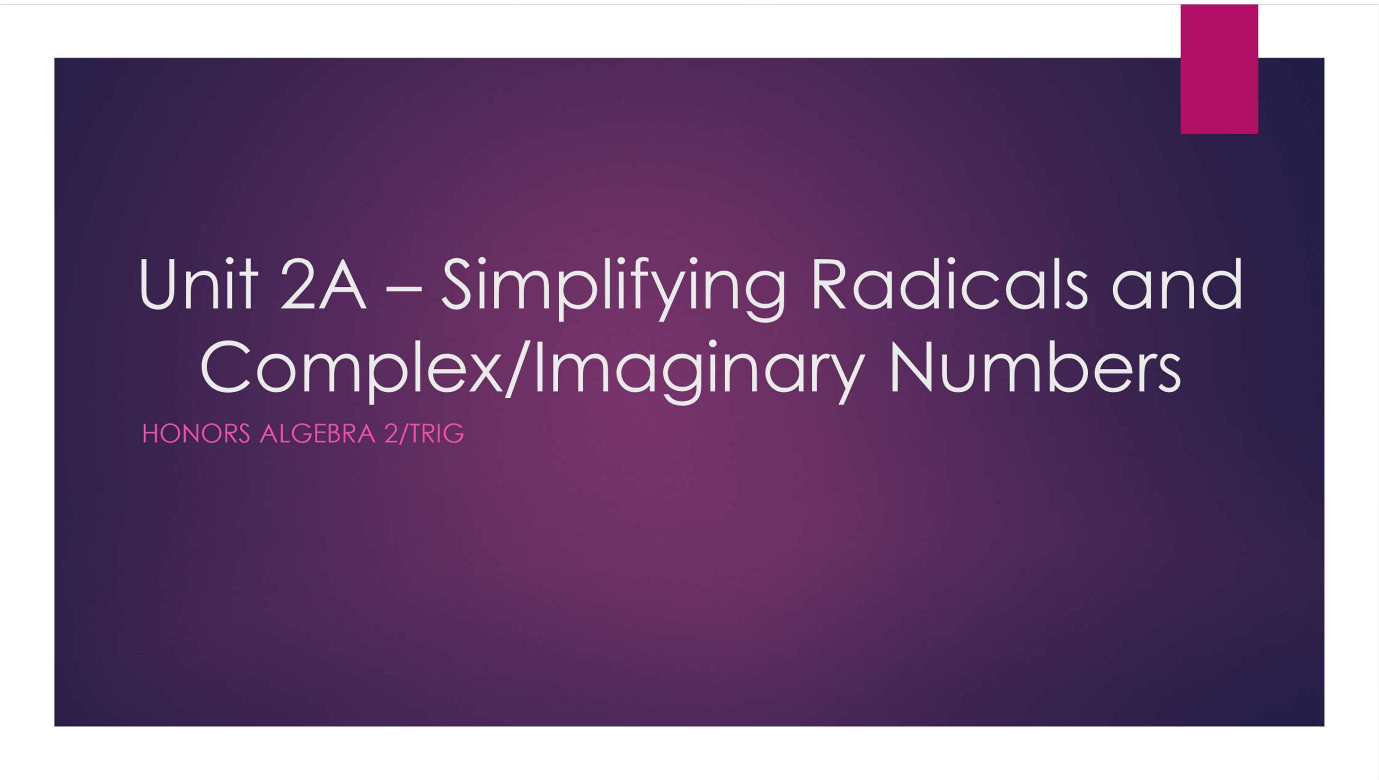
Notes B3

Thursday, February 7, 2019 8:05 AM

Day 3 - Imaginary and Complex

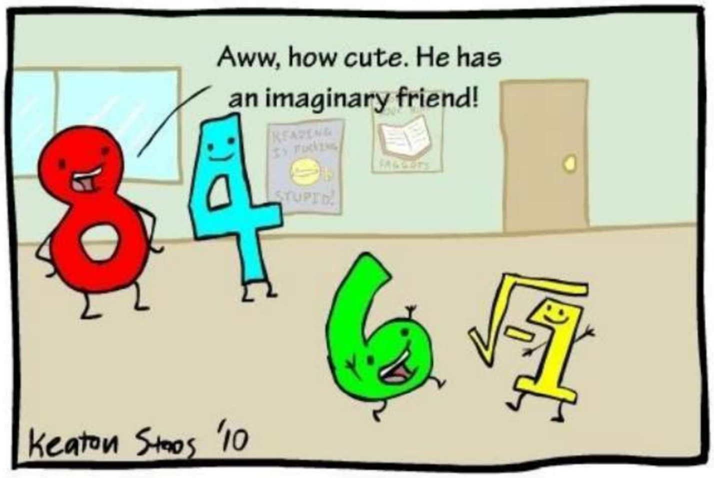


Unit 2A – Simplifying Radicals and Complex/Imaginary Numbers

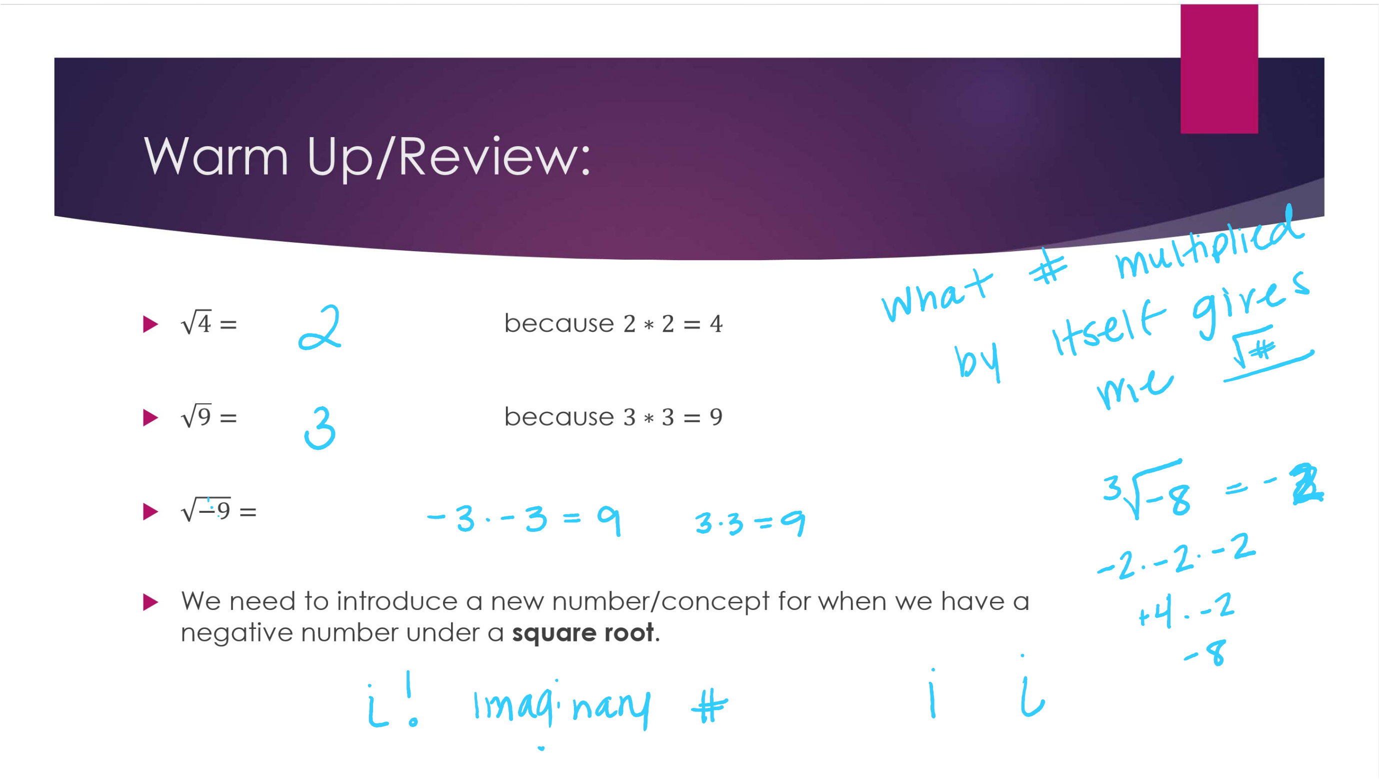
HONORS ALGEBRA 2/TRIG

Unit Goals

- Students will be able to:
  - Simplify Radicals of varying index
  - Add, Subtract, Multiply, and Divide Radicals
  - Rationalize the denominator when presented with radicals and radical expressions
  - **Identify and simplify complex and imaginary numbers**



Imaginary and Complex Numbers



Warm Up/Review:

- $\sqrt{4} =$ 2 because $2 * 2 = 4$
- $\sqrt{9} =$ 3 because $3 * 3 = 9$
- $\sqrt{-9} =$ $-3 \cdot -3 = 9$ $3 \cdot 3 = 9$
- We need to introduce a new number/concept for when we have a negative number under a **square root**.

i! imaginary # i i

what # multiplied by itself gives me $\sqrt{\#}$

$\sqrt[3]{-8} = -2$

$-2 \cdot -2 \cdot -2$

$+4 \cdot -2$

-8

Imaginary Numbers

- An Imaginary Number exists when there is a negative under a **square root**
  - We are allowed to have negative numbers under cubed roots, for example (think about it!)

$\sqrt[3]{-8} = -2$

- We use the letter i for imaginary numbers
  - $i = \sqrt{-1}$
  - $i^2 = -1$
  - $i^3 = -i$
  - $i^4 = 1$ →

Powers of i

$i^4 = i^2 \cdot i^2$
$= -1 \cdot -1$
$i^4 = 1$

$i^2 \cdot i = i^3$
$-1 \cdot i = i^3$
$-i = i^3$

ex.)

- $\sqrt{-5} = \sqrt{-1 \cdot 5}$
  $= \sqrt{-1} \cdot \sqrt{5}$
  $\boxed{i\sqrt{5}}$
- $\sqrt{-49} \rightarrow \sqrt{-1 \cdot 49}$
  $\sqrt{-1} \cdot \sqrt{49}$
  $i \cdot 7$
  $\boxed{7i}$
- $\sqrt{-12} \rightarrow \sqrt{-1 \cdot 12}$
  $\sqrt{-1} \cdot \sqrt{4 \cdot 3}$
  $\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{3}$
  $i \cdot 2 \cdot \sqrt{3}$
  $\boxed{2i\sqrt{3}}$

ex.)

- i^{34} $34 \div 4 = 8.5$ $\frac{1}{2} \rightarrow i^2 = \boxed{-1}$
- i^{72} $72 \div 4 = 18.0 \rightarrow \boxed{i}$
  no remainder!
- i^{357} $357 \div 4 = 89.25$ $\frac{1}{4} \rightarrow i^4 = \boxed{1}$
  $i^2 \cdot i^2$
  $-1 \cdot -1$

$4\overline{)34}$ = 8
32
2 ← i^2

$4\overline{)72}$ = 18
4↓
32
32
0

$4\overline{)357}$ = 89
32↓
37
36
1

Complex Numbers

$a+\sqrt{b}$

- When we combine imaginary and real numbers, we get a **complex number**
  - $a + bi$
  - Example: $5 + 3i$
- We can add and subtract these complex numbers like we have before!
  - Add the corresponding numbers and imaginary coefficients

Examples: $3+4i+5-3i$

- $(3 + 4i) + (5 - 3i) = \boxed{8+i}$
- $(2 + 3i) - (4 - 2i) = 2+3i-4+2i = \boxed{-2+5i}$
- $(2 - 4i) - (-2 + 3i) = 2-4i+2-3i = \boxed{4-7i}$
- $(5 + 6i) + 3(4 - 2i) = 5+6i+12-6i = \boxed{17}$
- $(1 - 11i) - (9i - 4) = 1-11i-9i+4 = \boxed{5-20i}$

Other Operations with Complex Numbers

MULTIPLYING AND DIVIDING

Multiplying Complex Numbers

- We will multiply complex numbers in the same manner as when we multiplied radicals (distribution, FOIL, etc.)
- It is important to remember the powers of *i* when we mulitply.
  - We cannot have any powers of *i* in our answer other than i and i^2

- $5i(-2+i) =$

$-10i + 5i^2$

$-10i + 5(-1)$

$-10i - 5$

$\boxed{-5-10i}$

$a \pm bi$

$(7-4i)(-1+2i) =$

$-7 + 14i + 4i - 8i^2$

$-7 + 18i - 8(-1)$

$-7 + 18i + 8$

$\boxed{1+18i}$

conjugates!

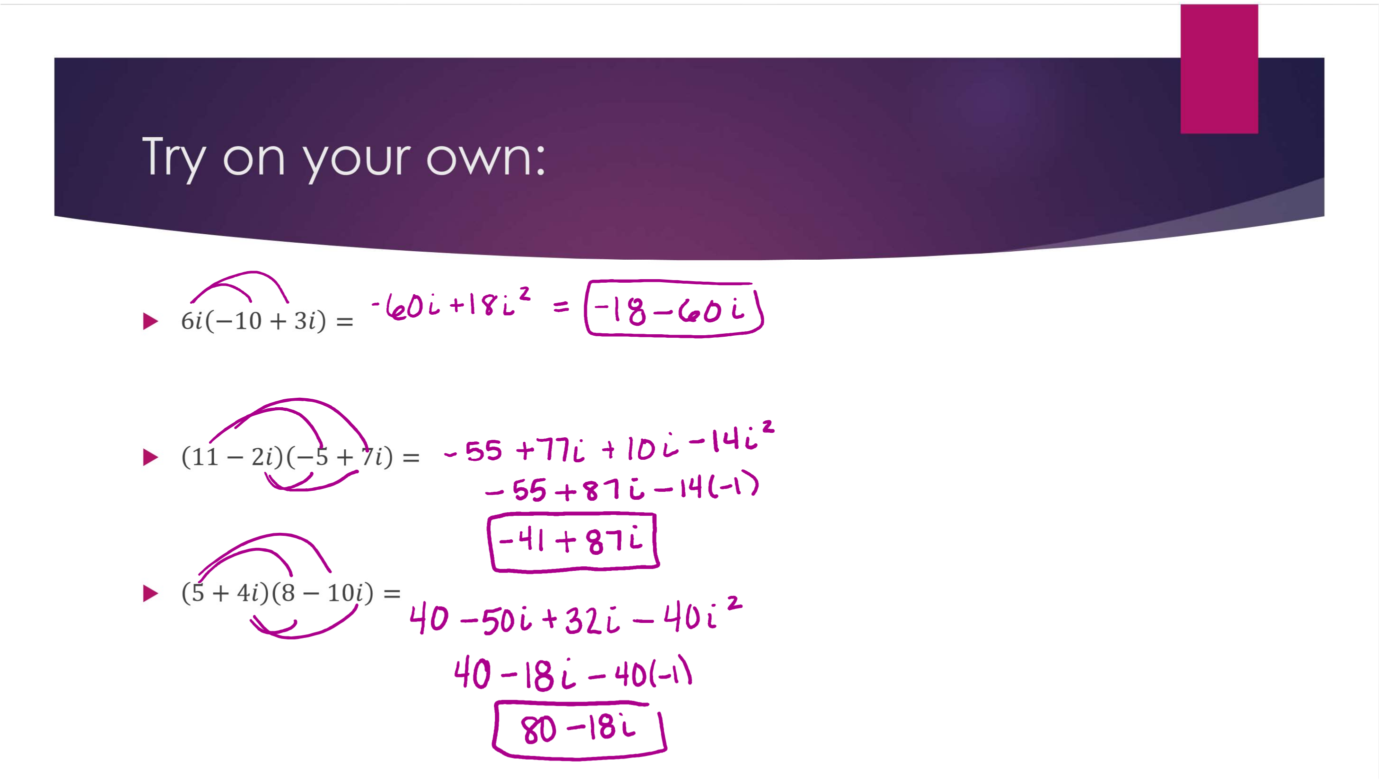
$(6+3i)(6-3i) =$

$36 \cancel{-18i + 18i} - 9i^2$

$36 - 9(-1)$

$36 + 9$

$\boxed{45}$



Try on your own:

- $6i(-10+3i) =$ $-60i+18i^2 = \boxed{-18-60i}$
- $(11-2i)(-5+7i) =$ $-55+77i+10i-14i^2$

  $-55+87i-14(-1)$

  $\boxed{-41+87i}$
- $(5+4i)(8-10i) =$

  $40-50i+32i-40i^2$

  $40-18i-40(-1)$

  $\boxed{80-18i}$

Division of Complex Numbers

- We CANNOT have complex numbers in the denominator.
  - In order to get rid of them, we can multiply both the numerator and denominator by the **conjugate** of the denominator.

$a+bi \leftrightarrow a-bi$

- $\frac{5+3i}{1-2i}\left(\frac{1+2i}{1+2i}\right) = \frac{5+10i+3i+6i^2}{5}$

$(1)^2-(2i)^2$

$1-4i^2$

$1+4$

$= \frac{5+13i+6(-1)}{5}$

$\boxed{= \frac{-1+13i}{5}}$

$-\frac{1}{5}+\frac{13}{5}i$

$a+bi$

- $\frac{4-2i}{3-3i}\left(\frac{3+3i}{3+3i}\right) = \frac{12+12i-6i-6i^2}{18}$

$(3)^2-(3i)^2$

$9-9i^2$

$9-9(-1)$

18

$= \frac{12+6i-6(-1)}{18}$

$= \frac{12+6i+6}{18}$

$= \frac{18+6i}{18} \div 6$

$\boxed{\frac{3+i}{3}}$

$\frac{3}{3}+\frac{1}{3}i$

$\boxed{1+\frac{1}{3}i}$

More Examples

- $\frac{5}{2+i}\left(\frac{2-i}{2-i}\right) = \frac{10-5i}{5} = \boxed{2-i}$

2^2-i^2

$4--1$

- $\frac{-5-3i}{4i}\left(\frac{i}{i}\right) = \frac{-5i-3i^2}{4i^2}$

$= \frac{-5i-3(-1)}{4(-1)}$

$\boxed{\frac{3-5i}{-4}}$

ex.) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

- $\frac{3+i}{} \left(\frac{3+i}{}\right) = \frac{9+3i+3i+i^2}{}$

- $\frac{10-i}{} \left(\frac{2-i}{}\right)$

- $\frac{3+i}{3-i}\left(\frac{3+i}{3+i}\right) = \frac{9+3i+3i+i^2}{10}$

  $3^2-(i)^2$

  $9--1$

  $= \frac{9+6i-1}{10}$

  $= \frac{8+6i}{10} \div 2$

  $= \boxed{\frac{4+3i}{5}}$

- $\frac{10-i}{2+i}\left(\frac{2-i}{2-i}\right)$ $\boxed{\frac{3-5i}{-4}}$

  $= \frac{20-10i-2i+i^2}{5}$

  $= \frac{20-12i-1}{5}$

  $= \boxed{\frac{19-12i}{5}}$

  $\frac{19}{5} - \frac{12}{5}i$

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#47-60 1st Column

CLASSWORK/HOMEWORK

~~QUEST TOMORROW~~ Quest Tuesday